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Effectiveness-robustness objectives in MTMD system design: An evolutionary optimal design methodology

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SUMMARY

The main contribution pursued in this investigation is to propose a generic optimal methodology for designing multiple tuned mass damper (MTMD) systems to efficiently suppress the response of uncertain structures to harmonic excitations. This methodology comprises quantitative indexing of MTMD effectiveness and robustness as performance criteria, implementing a two-objective optimization and presenting solutions as optimal set of non-dominated designs. Within this three-step framework, utilization of a new index of robustness is proposed, advantages of exploiting multi-objective genetic algorithm are demonstrated, and the trade-off between effectiveness and robustness is visualized via Pareto-optimal solution fronts. A systematic stochastic procedure has been also proposed and carried out to provide validation and further insight into the robustness concept. Although a uniformly distributed MTMD (UMTMD) model is utilized for the illustration purpose, the formulation of the objective functions are generic, while at the same time, simple and convenient to modify according to any desired assumptions; hence, the proposed methodology is applicable to a broad range of problems. To demonstrate this versatility, this methodology is employed to optimize an irregular MTMD (IMTMD) without restrictive assumptions and to show its ability of handling design spaces with rather large dimensionality and complexity. Comparison of the results obtained from optimal effectiveness–robustness design of irregular and uniform MTMDs reveals the performance advantage of IMTMD over UMTMD design. Copyright © 2008 John Wiley & Sons, Ltd.

KEY WORDS: multiple tuned mass damper (MTMD); optimal design methodology; effectiveness; robustness; multi-objective GA; Pareto-optimal solutions

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1. INTRODUCTION

The utilization and optimization of (multiple) tuned mass damper ((M)TMD) systems as effective passive control devices in engineering structures have been theoretically well established in the literature [1–20] and extensively implemented in practice [21].

The mainstream trend of research [3,4,6–8,10–18] on MTMD systems has been directed toward performance optimization (optimal tuning) with focus on improving effectiveness in terms of the amount by which the MTMD suppresses structural vibrations. These studies enable designers to achieve optimal performance only under nominal conditions that these passive controllers are designed to operate. However, in practice, structural properties and dynamic characteristics are subjected to various types of uncertainties. These issues are manifested through uncertainties in the mass, damping and stiffness of structural models originating from a variety of sources, such as inadequate modeling of the boundary conditions at the structural joints, effects of non-structural elements, degradation due to aging, and fluctuations in structural mass (e.g. traffic on a bridge), as well as uncertainties in the member capacities, yield strengths, etc. [9]. Furthermore, the dynamic characteristics of structures change under earthquake or wind excitations. Some structures have uncertain nonlinear properties even in the small amplitude range due to the contribution of secondary members. Therefore, in practical design problems, these uncertainties exist and if neglected can lead to drastic reduction of performance due to mistuning. To avoid such undesired MTMD performance degradation, the modeling uncertainties and estimation errors must be accounted for, i.e. MTMD robustness to uncertainties must be treated as a design concern. As is the most significant, the estimation error in the natural frequency of the structure represents the source of modeling uncertainty against which robustness is considered throughout this paper.

Unlike most of the previous works where optimization is based on effectiveness and, robustness is either not considered [10,12,14–17] or only checked after the MTMD has been designed [6–8,11,13,18], in the present investigation both criteria would play equally important roles in quantifying MTMD performance, indeed each will index separate objectives. Of course, the concept of robustness shall be addressed more thoroughly to compensate for the lack of available quantitative formulations from previous works, as will be discussed hereafter. Toward the scope of this paper in developing an optimal design methodology for MTMD systems, three steps should be taken as explained in what follows.

1.1. Quantitative indexing of performance criteria

The effectiveness, as the nominal performance, can be most generally indexed by the H_2 and/or H_∞ norms of the transfer function of the structure-MTMD dynamic system from disturbance input to the regulated output [14]. Zuo and Nayfeh [16] have compared mini max (maximization of the minimal damping), H_2 and H_∞ methods for effectiveness optimization of MDOF TMDs and multiple SDOF TMDs. In the present investigation, the effectiveness will be evaluated using the worst case H_∞ concept, commonly referred to as the maximum dynamic magnification factor (max DMF) of the structure-MTMD system in the literature, with the objective being its minimization (min max DMF). While criteria for quantitative evaluation of effectiveness have been well established, the situation reveals to be completely different for the case of robustness.

Although well identified as a performance criterion, robustness has merely received qualitative comments from most researchers. Yamaguchi and Harnpornchai [6] associated the concept of robustness with the qualitative behavior of the max DMF to changes in the natural

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frequency error; Kareem and Kline [8] interpreted this concept in a similar fashion; Rana and Soong [11] showed that the effect of detuning in TMD parameters become less significant as the structural damping and/or mass ratio is increased; Li [13] pointed out the flatness of response curves as the characteristic of more robust MTMD systems; Abe and Fujino [7], in an attempt to go slightly beyond qualitative description, proposed the reserve bandwidth criterion; and Han and Li [18] stated that increasing the optimum frequency spacing leads to enhancement of robustness. In all these investigations following the mainstream trend of nominal effectiveness optimization, robustness is qualitatively checked and commented on only after the MTMD system has been designed. On the other hand, there are a number of investigations that have focused on uncertainty, through a probabilistic and/or reliability-based perspective. Such studies have been based on minimizing the expected value of the mean square response [5] or minimizing the failure probability [9] over the domain of all possible values of the uncertain system parameters, usually for single TMDs. However, there seems an apparent gap between these two trends of research on MTMD systems design; in other words, there are few, if any (see Li and Ni [19] and also Section 7.2), investigations in the literature that have optimized effectiveness along with simultaneous quantitative consideration of robustness.

To the authors' knowledge, robustness has been never indexed quantitatively and treated as an optimization objective along with effectiveness in a two-objective evolutionary optimization. This paper is intended to establish this two-objective optimization methodology using the proposed robustness index $\Re$ (see Section 3.2) within a generic framework.

1.2. Implementation of the optimization problem

MTMD effectiveness and robustness are in conflict and cannot be properly combined into a single objective, as shown by the authors in [20]. Considering this conflicting behavior of these two optimization criteria with variation of MTMD design parameters, in this paper, it is proposed to formulate the optimal design of MTMD systems as a two objective optimization problem. A genetic algorithm (GA) has been implemented (see Section 4) due to its appealing ability to converge to the non-dominated set of solutions rather than a single optimal solution.

1.3. Presentation of the results

It will be shown that the set of solutions, obtained from the optimization problem, can be conveniently presented through the concept of Pareto optimality [22]. To discuss and validate the results, an analysis procedure is proposed in Section 6 that would provide further insight into the proposed robustness concept by its unification with the probabilistic perspective. This discussion would also yield a possible preference utility for design selection from the entire set of non-dominated solutions.

2. STRUCTURE-MTMD SYSTEM

The optimal design methodology for MTMD system design that will be proposed in this paper is generic in the sense that it is applicable to any desired model under arbitrary assumptions. However, for the purpose of illustration, let us consider a structure-MTMD model as shown in Figure 1. The structure is represented by its mode-generalized system in the specific vibration mode being controlled, and together with the MTMD form a $(n+1)$-DOF system.

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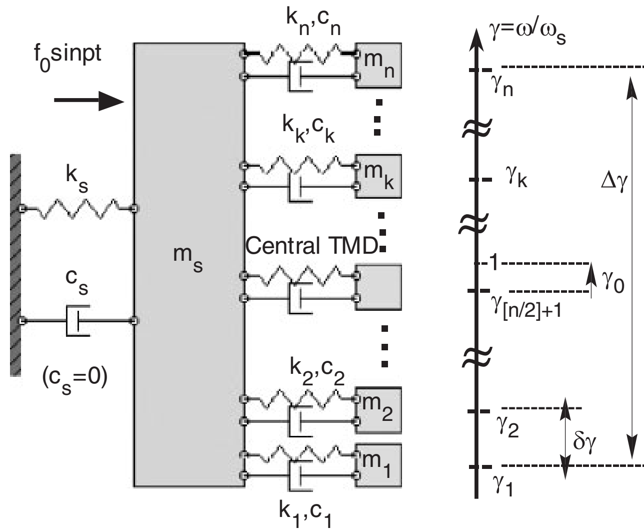

Figure 1. Structure-MTMD system model and assumptions.

For the system shown in this figure, the DMF of the structural response is derived as [6]

$$\mathrm{DMF} = \frac{1}{\sqrt{\mathrm{Re}^2(Z) + \mathrm{Im}^2(Z)}} \tag{1}$$

where

$$\mathrm{Re}(Z) = 1 - \gamma^2 - \sum_{k=1}^{n} \frac{\mu_k \gamma^2 [\gamma_k^2(\gamma_k^2 - \gamma^2) + (2\zeta_k \gamma_k \gamma)^2]}{(\gamma_k^2 - \gamma^2)^2 + (2\zeta_k \gamma_k \gamma)^2}$$

$$\mathrm{Im}(Z) = 2\zeta_s \gamma + \sum_{k=1}^{n} \frac{2\mu_k \zeta_k \gamma_k \gamma^2}{(\gamma_k^2 - \gamma^2)^2 + (2\zeta_k \gamma_k \gamma)^2}$$

with the following parameters: γ is the ratio of external force frequency ω to the structural natural frequency ω_s; ζ_s is the structural damping ratio; n is the total number of TMDs with the MTMD also referred to as a nTMD; μ_k is the mass ratio of the kth TMD defined as m_k/m_s; γ_k is the frequency ratio of the kth TMD defined as ω_k/ω_s; ζ_k is the damping ratio of the kth TMD defined as $c_k/(2m_k\omega_k)$.

In order to further facilitate the illustration of the concepts regarding the optimal design methodology, a uniformly distributed MTMD (UMTMD) system on an undamped structure is considered under the following assumptions:

(1) $\mu_k = \mu/n =$ cte: equal distribution of the total TMD mass ratio $\mu = 0.01$.
(2) $\gamma_k - \gamma_{k-1} = \delta\gamma_k = \Delta\gamma/n =$ cte: uniform distribution of the frequency ratio spacings $\delta\gamma_k$ with a total frequency range $\Delta\gamma$.
(3) $\zeta_k = \zeta =$ cte: equal damping ratios for all TMDs.
(4) $\gamma_0 = 0$: zero offset of the central TMD frequency ratio. This additional simplification of zero offset will be shown to cause only a negligible negative effect on the UMTMD optimality (see Section 6).

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Accordingly, the only independent parameters to be determined for defining the UMTMD would be the number n of TMDs, the damping ratio ζ and the frequency range $\Delta\gamma$. These assumptions, hence, make the nTMD design space shrink to the 2-dimensional $\zeta - \Delta\gamma$ space, which can be conveniently visualized. Although useful for the purpose of illustration and visualization, these assumptions would restrict the optimal UMTMD performance confronted to the optimal irregular MTMD (IMTMD). Hence, in Section 7, the methodology would be employed to design an IMTMD free of these restrictions.

3. PERFORMANCE CRITERIA

In this section the two objectives of optimization will be specified. These two objectives are the two aforementioned criteria of performance, evaluated through appropriate indices that would be presented.

3.1. Effectiveness indexed by max DMF

The most broadly accepted and applied objective of effectiveness optimization is the minimization of the maximum DMF (min max DMF) of the structure-MTMD system. This might involve the minimization of the worst case displacement DMF or acceleration DMF under external disturbing force or ground excitation (min max DDMF; min max ADMF) [13]. In this paper, the displacement DMF (DDMF) is considered and referred to as DMF of which peak value is calculated from Equation (1) over frequency as the first minimization objective function utilized in the GA optimization (similarly, for any other problem, the peak frequency response can be used).

Figure 2 compares the DMF frequency response of three 5TMD systems with fixed damping and different frequency ranges. As can be concluded from this figure, an optimal frequency range $\Delta\gamma_{opt}$ exists that minimizes the max DMF response for any fixed values of TMD damping

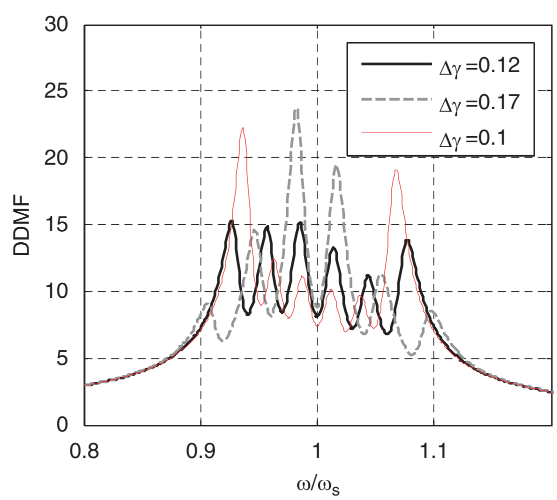

Figure 2. DDMF response for MTMD with $\zeta = 0.01$ and $n = 5$.

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ζ and total number n. Accordingly, a min max DMF $\zeta - \Delta\gamma$ contour exists for any nTMD as shall be later depicted as the lower bounds of design in Figure 8(a) for $n = 5$, 7 and 11.

Detailed studies on the influence of MTMD design parameters (frequency range, TMD damping and TMDs total number) on effectiveness according to the max DMF index are available in the literature (e.g. see Reference [6]).

3.2. Robustness indexed by $\Re$

The flatness of the max DMF curve versus error in natural frequency is a good qualitative index of the robustness of a MTMD-system in the presence of uncertainty in the natural frequency of the structure. To derive a quantitative index from this concept, it was proposed [20] to measure robustness through an averaging of angles α_i^{R} and α_i^{L} defined on this curve, as shown in Figure 3. Applying the axes scaling and definition of angles shown in this figure, the average angles of robustness with respect to positive and negative (right and left) errors would be expressed as follows:

$$\begin{aligned}\bar{\alpha}_{\mathrm{R}} &= \frac{1}{N}\sum_{i=1}^{N}\cot^{-1}\left\{\frac{1}{e_i}[\max \mathrm{DMF}_{\mathrm{dB}}(e_i) - \max \mathrm{DMF}_{\mathrm{dB}}(0)]\right\} \\ \bar{\alpha}_{\mathrm{L}} &= \frac{1}{N}\sum_{i=1}^{N}\cot^{-1}\left\{\frac{1}{e_i}[\max \mathrm{DMF}_{\mathrm{dB}}(-e_i) - \max \mathrm{DMF}_{\mathrm{dB}}(0)]\right\}\end{aligned} \tag{2}$$

where e_i is the relative error percentage, which in the summation varies from zero to e_{R}, e.g. 5 or 10%, in both directions; N is the number of points to which the intervals $[0,e_{\mathrm{R}}]$ and $[-e_{\mathrm{R}},0]$ are discretized; and, max $\mathrm{DMF}_{\mathrm{dB}}$ is the value of max DMF expressed in dB units as a function of the error percentage. Applying the dB unit conversion and also normalizing by the right angle of

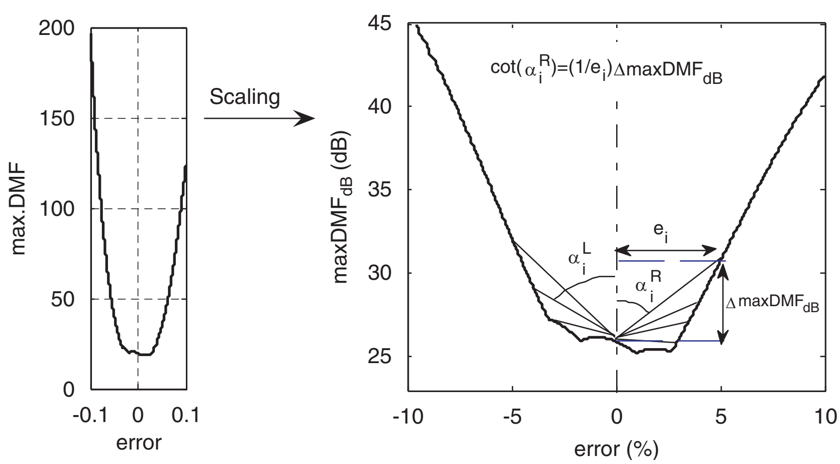

Figure 3. Scaling and definition of angles.

$\pi/2$ one can obtain the left and right robustness as

$$\Re_{\mathrm{R}} = \frac{2}{\pi N}\sum_{i=1}^{N}\cot^{-1}\left[\frac{20N}{e_{\mathrm{R}}.i}.\log_{10}\left(\frac{\max \mathrm{DMF}(e_{\mathrm{R}}.i/N)}{\max \mathrm{DMF}(0)}\right)\right]$$
$$\Re_{\mathrm{L}} = \frac{2}{\pi N}\sum_{i=1}^{N}\cot^{-1}\left[\frac{20N}{e_{\mathrm{R}}.i}.\log_{10}\left(\frac{\max \mathrm{DMF}(-e_{\mathrm{R}}.i/N)}{\max \mathrm{DMF}(0)}\right)\right] \tag{3}$$

It is reasonable to define the overall robustness $\Re$ as the minimum of $\Re_{\mathrm{L}}$ and $\Re_{\mathrm{R}}$. Since the output of $\cot^{-1}$ function, in Equations (2) and (3), is assumed to be in the range $[0,\pi]$, the numerical values of $\Re$, $\Re_{\mathrm{L}}$ and $\Re_{\mathrm{R}}$ would be restricted to the range (0,2), with a numerical value of unity representing relative insensitivity of effectiveness to error in ω_s. Numerical values less or greater than unity refer to decreasing or increasing behaviors of effectiveness against error, respectively. It is also interesting to note that since the offset frequency of the central TMD in the MTMD model used here was set to be zero, the positive error in the natural frequency of the structure makes the MTMD more effective. This leads to a greater value of $\Re_{\mathrm{R}}$ compared with the value of $\Re_{\mathrm{L}}$, making the latter dominant for indexing the overall robustness, which often would be less than unity.

A continuous version of the robustness-pair relations (3) can also be constructed as

$$\Re_{\mathrm{R}} = \frac{2}{\pi e_{\mathrm{R}}}\int_0^{e_{\mathrm{R}}}\cot^{-1}\left[\frac{20}{e}.\log_{10}\left(\frac{\max \mathrm{DMF}(e)}{\max \mathrm{DMF}(0)}\right)\right]\mathrm{d}e$$
$$\Re_{\mathrm{L}} = \frac{2}{\pi e_{\mathrm{R}}}\int_0^{e_{R}}\cot^{-1}\left[\frac{20}{e}.\log_{10}\left(\frac{\max \mathrm{DMF}(-e)}{\max \mathrm{DMF}(0)}\right)\right]\mathrm{d}e \tag{4}$$

The influence of the frequency range $\Delta\gamma$ on the robustness of a 5TMD with 0.01 TMD damping is visualized in Figure 4(a). It is most important to note that this $\Re$-curve has a minimum and a maximum. The fact that the minimum almost occurs at the frequency range

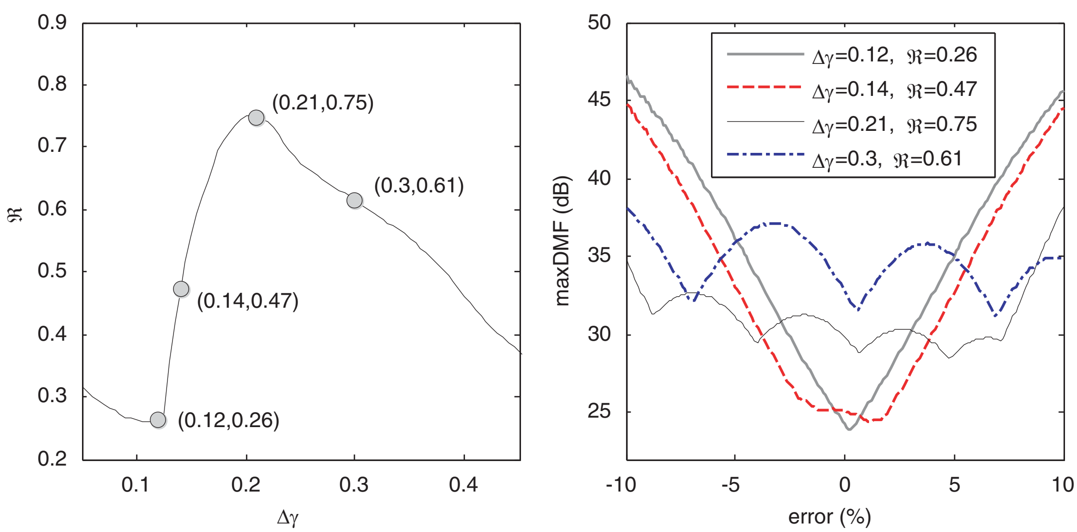

Figure 4. max DMF versus nat. freq. error with $e_{\mathrm{R}} = 10\%$ for a 5TMD ($\zeta = 0.01$).

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that minimizes max DMF reveals the conflicting behaviors of robustness and effectiveness. On the other hand, the existence of the maximum enables us to obtain max$\Re$ $\zeta - \Delta\gamma$ contours for nTMDs to be later plotted in Figure 8 as the upper bounds of design along with the previously mentioned min max DMF $\zeta - \Delta\gamma$ contours. Figure 4(b) depicts the effectiveness behaviors of four MTMDs chosen from Figure 4(a) to changes in relative error in the structural frequency ω_s. As can be concluded, the numerical value of the proposed index is in perfect harmony with the qualitative judgment on robustness.

The effect of MTMD design parameters on robustness according to this index is thoroughly investigated and compared with their respective influence on effectiveness in Reference [20]. It was shown that while there exists a specific frequency range for maximizing robustness, increasing number of TMDs and the damping ratio always enhances robustness. It was also shown that single-objective optimization, targeting only effectiveness, leads to an MTMD, which is (slightly) more effective than an optimal single TMD with the non-justifiable disadvantage of being less robust.

4. OPTIMIZATION IMPLEMENTATION

As mentioned previously, maximization of effectiveness and robustness are conflicting objectives. Hence, a compromise is required in choosing MTMD design parameters, i.e. $\Delta\gamma$, ζ and n. This compromise can be conveniently chosen from the entire set of non-dominated solutions obtained from two-objective optimization based on the designer's preferences. As will be discussed, in this research, optimization is based on GA implementation because of its advantages. Indeed, the proposed methodology mostly owes its simplicity and flexibility to this exploitation of evolutionary heuristics.

GA is a computational representation of natural selection, inspired by the concept of higher survival chance of the fittest individual in its environment through successive generations to find the optimal design among the others. Detailed treatment of GA methods, concepts and operators can be found in the literature [23] as it has proved its efficiency in handling various problems, including optimization of passive and active structural control, e.g. [24,25]. The main advantages of GAs can be summarized in that they do not require and do not depend on gradient information; they use a population of design points and randomly utilize information from each generation to the subsequent one; and, there is a potential to make the population converge to the Pareto-optimal set. The latter property is a crucial appeal of GA in multi-objective optimization problems that enables us to use GA in combination with a Pareto-set filter [26] to obtain a near approximation of the entire set of non-dominated solutions.

A point X^* in the feasible design space S is called Pareto optimal if there is no other point X in the set S that improves (decreases) at least one objective function without degrading (increasing) another one [26]; or, mathematically stated for two objectives

$$\{X^* \in S: \text{ Pareto optimal}\} \Leftrightarrow \left\{ \begin{array}{l} X \in S\text{: } f_1(X) \leqslant f_1(X^*),\ f_2(X) \leqslant f_2(X^*) \\ \text{with at least one inequality} \end{array} \right\} \tag{5}$$

Based on this definition a Pareto-set filter can be incorporated in the GA optimizer as a sub-function, coded as shown in Figure 5, to guard against loosing potential Pareto-optimal solutions in a particular generation. In addition to the population at each generation, the GA optimizer stores a Pareto set, which contains all non-dominated solutions from the beginning up to the previous iteration. At the end of each iteration, the new population is added to the Pareto

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```
Subfunction {pareto}=filter_pareto({solutions},f1,f2)
This subfunction extracts the Pareto set out of the entire set of solutions. The main GA
optimizer in each iteration calls it as: {pareto }=filter_pareto({pareto}+{pop},f1,f2)

{pareto}={solutions}
for i=1 to size({solutions}
   X=solutions(i);
   {set}={pareto}-{X};
   for j=1: size({set})
      Y=set(j);
      if (f1(X)>=f1(Y)) & (f2(X)>=f2(Y)) & not(f1(X)=f1(Y) & f2(X)=f2(Y))
         X is dominated by Y, so must be removed from the Pareto set
         {pareto}={set};
      endif
   endfor
endfor
```

Figure 5. Pseudo-code of the Pareto-set filter.

Table I. GA specifications.

Algorithm type	Two-branch tournament
Minimization objectives	max DMF & $1/\Re$
Population size	1000
Number of generations	50
Cross over	
Type	Two point
Prob.	0.7
Mutation probability	0.02

set and this entire set of solutions undergoes a non-dominated check within the filter, the non-dominated solutions are filtered out and the Pareto set is updated.

In this investigation a two-branch tournament algorithm, based on this feature of GA, is implemented for the specific problem in hand. For detailed treatment and concepts concerning general use of two-branch tournament GA in multi-objective optimization, Reference [27] might be consulted. Reference [25] involves a similar application of this algorithm to an active structural control problem. The specifications of the GA implementation in the present study are listed in Table I.

5. RESULTS

In this investigation, the design space for the optimization problem is ζ–$\Delta\gamma$ and the criterion space is $1/\Re$-max DMF. The results of the GA two-objective optimization are presented in these two spaces for different total numbers of TMDs in Figures 6 and 7. For better visualization of the mapping carried out by the objective functions from the design space to the criterion space, a number of points on the 5TMD optimal curves are labeled.

Figure 7 shows also that by increasing the total number of TMDs it is possible to achieve higher robustness while maintaining a constant effectiveness, and vice versa.

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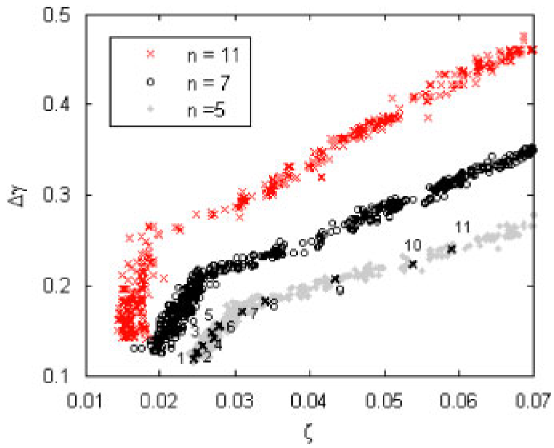

Figure 6. Pareto sets of points in the design space.

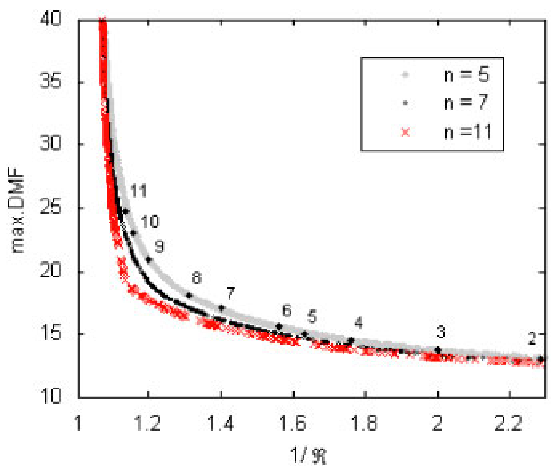

Figure 7. Pareto sets of solutions in the criterion space.

In Reference [20], the authors proposed a systematic methodology to select the UMTMD frequency range for any fixed set of values for the TMD damping and total number. This methodology confined the selection of the frequency range between the lower bounds of maximum effectiveness (min max DMF ζ–$\Delta\gamma$ contours) and the upper bound of maximum robustness (maxζ–$\Delta\gamma$ contours). Now we can add the Pareto set of designs to the design diagrams in order to more accurately guide the designer's choice to the non-dominated solutions (Figure 8).

6. DISCUSSION

In this section, the results (optimal set of solutions) shall be verified through a stochastic analysis. This analysis will offer a different perspective toward the concept of robustness that

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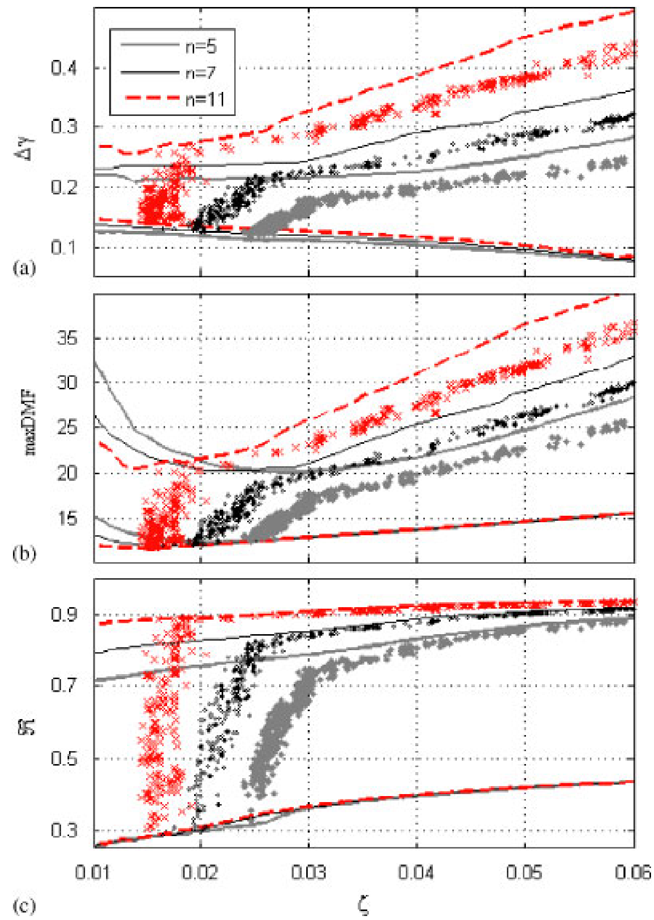

Figure 8. (a) Pareto-optimal points in design space within design bounds. (b) Corresponding effectiveness of Pareto-optimal solutions within design bounds. (c) Corresponding robustness of Pareto-optimal solutions within design bounds.

would prove to be consistent to the previous interpretation. This procedure, as illustrated in Figure 9, involves choosing a specific MTMD design to be employed, applying a prescribed probability distribution (with appropriate density function) to the estimation error in the natural frequency of the uncertain SDOF structure, and, finally, studying the resulting distribution of the structure-MTMD max DMF response.

Let us carry out the aforementioned analysis on an adequate number of MTMDs chosen from the Pareto-optimal solutions from the previous section and study the resulting distribution of their max DMF responses, especially their means and standard deviations. In this investigation, a symmetric normal distribution of estimation error in structural frequency with a coefficient of variation of 3% through 10 000 samplings is applied to the chosen Pareto-optimal solutions.

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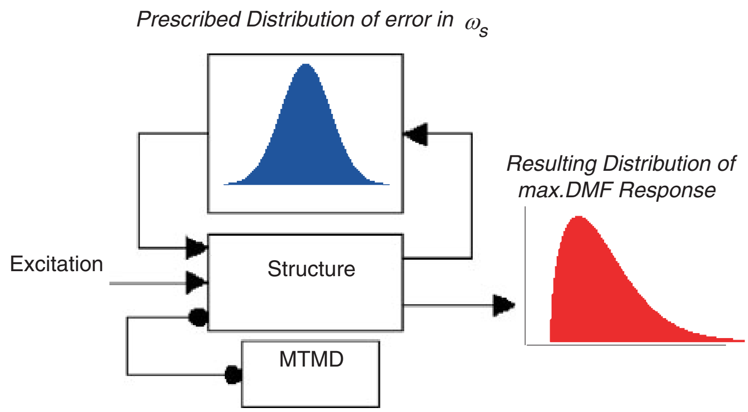

Figure 9. max DMF distribution analysis.

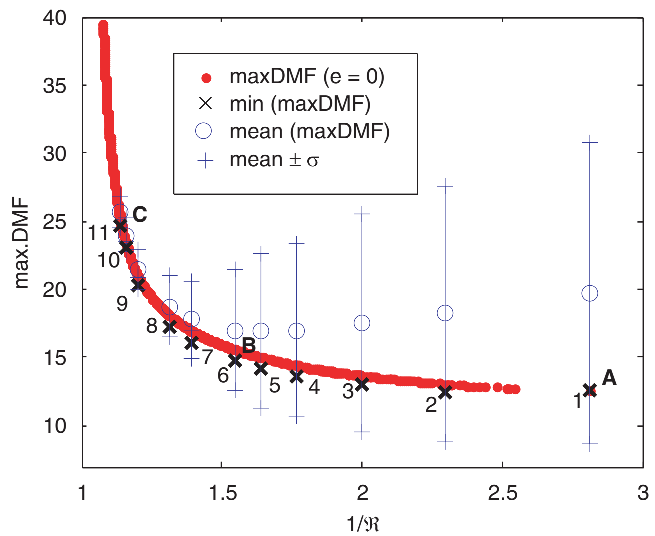

Figure 10. Optimal 5TMD solutions chosen for the analysis.

Figure 10 shows the chosen optimal 5TMD solutions (these selected points are the same as those shown in Figures 6 and 7) on the Pareto front in the criterion space and visualizes the mean value and the standard deviation of the corresponding uncertain max DMF distribution resulting from each through the analysis. Similar figures (Figures 11 and 12) are also presented for the case of 7 and 11 number of TMDs. In these figures the mean value of the max DMF response for each selected design solution is shown and two-side bracketed pictorially by the corresponding standard deviation. It should be noted that this illustration of the standard deviation is for visualization and neither the max DMF distribution is symmetrical about the mean, nor does the mean value minus the standard deviation occur in the distribution. Indeed, the minimum value in the max DMF distribution at each robustness corresponds approximately to the non-dominated

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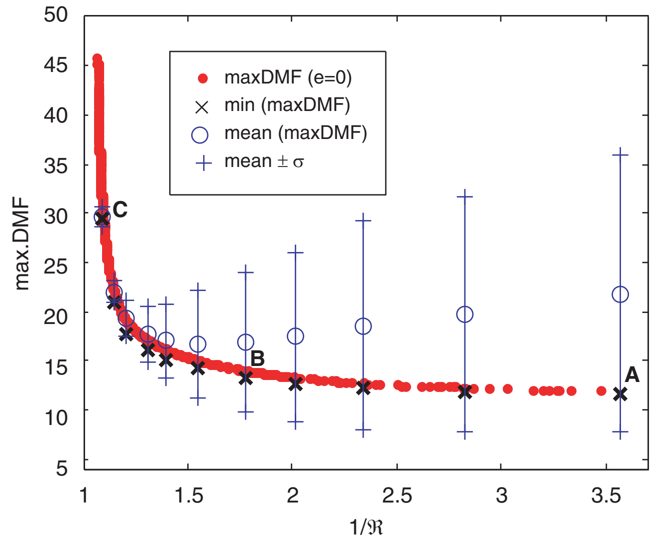

Figure 11. Optimal 7TMD solutions chosen for the analysis.

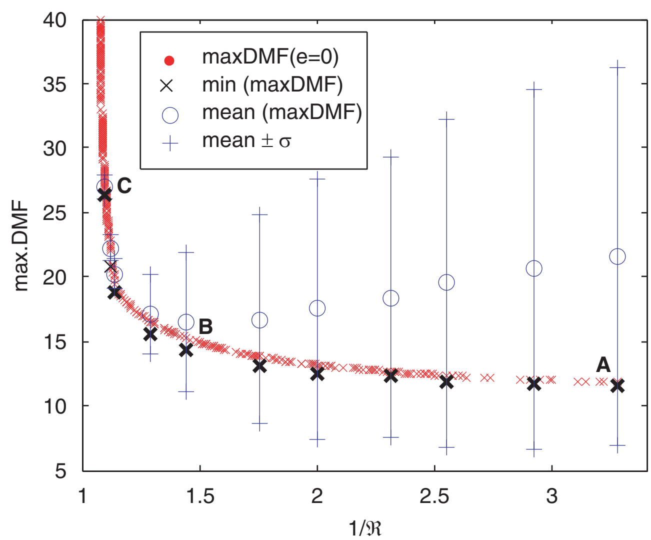

Figure 12. Optimal 11TMD solutions chosen for the analysis.

design point, i.e. the zero-error point, with a negligible difference due to the justifiable simplification assumption of zero offset frequency (assumption 4 in Section 2).

On each of the Pareto fronts depicted in Figures 10–12 three representative points are labeled with A, B and C. Points A and C correspond, respectively, to optimal effectiveness and robustness designs, while point B represent a judicious compromise (later it will be shown how to reach this compromise using a preference criterion). Table II lists the numerical values of

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Table II. Comparison of different MTMD designs.

N	Point	ζ	$\Delta\gamma$	max DMF	$\Re$	Min	Std	Mean
5	A	0.0246	0.1183	12.53	0.3553	11.94	10.971	19.71
	B	0.0280	0.1544	15.61	0.6467	14.70	4.435	16.96
	C	0.059	0.2401	25.26	0.8802	24.74	1.130	25.72
7	A	0.0166	0.1298	12.028	0.28	11.69	14.114	21.83
	B	0.0235	0.1656	15.02	0.645	14.207	5.44	16.67
	C	0.058	0.314	29.39	0.914	28.27	1.01	29.55
11	A	0.0151	0.1408	11.82	0.3049	11.53	14.63	21.52
	B	0.0166	0.1896	15.23	0.6944	14.35	5.397	16.51
	C	0.037	0.33	26.84	0.9174	25.53	0.94	27

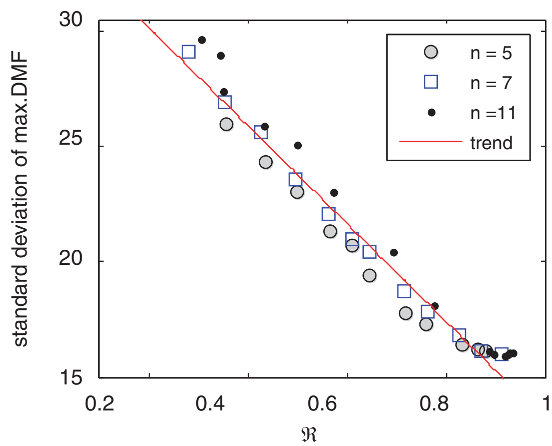

Figure 13. Standard deviation of max DMF distribution and robustness.

MTMD parameters, performance criteria and max DMF distribution indices (minimum, standard deviaton and mean) of these points for comparison.

As previously mentioned, the main purpose of this analysis is to study the variations of the standard deviation and mean value of the max DMF distribution, as shown in the Figures 13 and 14, due to different MTMD design decisions (each assigned a different value of the robustness index $\Re$).

The standard deviation is a measure of the spread or dispersion of the max DMF distribution due to sensitivity to uncertainty; and, thus, is correlated to the robustness of the MTMD as shown in Figure 13.

Figure 14 depicts the variation of the mean value of the max DMF response with different MTMD design decisions and their corresponding different values of robustness. An interesting observation is the existence of an optimum point on the mean max DMF trend curve, which is on the Pareto front but corresponds neither to the min max DMF nor to the max $\Re$ criterion. This motivates the suggestion of the following preference criterion for selection of a design from the entire set of Pareto-optimal solutions

$$\min_{\text{MTMD parameters}\in\text{Pareto}} \text{mean}(\max \text{DMF}) \tag{6}$$

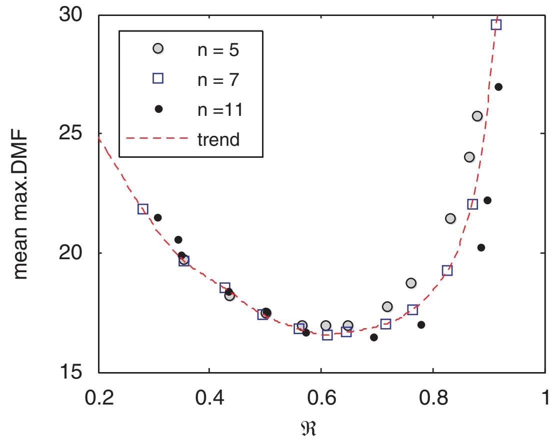

Figure 14. Mean max DMF versus robustness.

According to this min mean(max DMF) criterion the preferred designs among the Pareto fronts of Figures 10–12 have numerical values of $\Re$ equal to the minimum of the curve presented in Figure 14 (approximately 0.6–0.7). The chosen points B in Table II satisfy this criterion, hence, can be considered judicious compromises.

7. EXTENSIONS

As emphasized in this paper, the most important characteristics of the proposed methodology are its simplicity and its broad range of applicability or generality. Furthermore, the methodology can be conveniently modified and easily extended as would be concisely illustrated in what follows.

7.1. Modification of objective functions

Following the methodology framework, the designer might conveniently modify each of the objective functions or performance indices. As a case in point, the robustness definition (4) of Section 3.2 along with the probabilistic discussion in Section 6 motivates the general definition of $\Re$ considering utilization of a general desired probability distribution on $(-\infty, \infty)$ instead of a uniform error in the interval $[-e_{\mathrm{R}}, e_{\mathrm{R}}]$. For symmetrical normal distribution with standard deviation σ, the continuous generalized version of the robustness-pair relations can be obtained by weighted averaging as

$$\begin{aligned}\Re_{\mathrm{R}} &= \frac{1}{\sigma}\left(\frac{2}{\pi}\right)^{3/2}\int_0^{\infty} \exp\left(\frac{-e^2}{2\sigma^2}\right)\cot^{-1}\left[\frac{20}{e}.\log_{10}\left(\frac{\max \mathrm{DMF}(e)}{\max \mathrm{DMF}(0)}\right)\right]\mathrm{d}e \\ \Re_{\mathrm{L}} &= \frac{1}{\sigma}\left(\frac{2}{\pi}\right)^{3/2}\int_0^{\infty} \exp\left(\frac{-e^2}{2\sigma^2}\right)\cot^{-1}\left[\frac{20}{e}.\log_{10}\left(\frac{\max \mathrm{DMF}(-e)}{\max \mathrm{DMF}(0)}\right)\right]\mathrm{d}e\end{aligned} \tag{7}$$

with $1/\Re$ as the robustness minimization objective (a convenient discretization on a sufficiently large but finite interval for error might be used instead).

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As another possibility, on the other hand, instead of utilizing $1/\Re$ with $\Re$ defined as (4) or (7), the standard deviation of max DMF distribution (see Section 6) resulting from a prescribed structural frequency error distribution might be adopted as robustness minimization objective. Under uniform distribution of structural natural frequency error in $[-e_R, e_R]$, this criterion would be simply written as

$$\min_{\text{MTMD parameters}} \text{rms}(\max \text{DMF}(e) - \max \text{DMF}(0)) \tag{8}$$

with the rms (root mean square) function.

7.2. Employment of the methodology on arbitrary model and assumptions

To demonstrate the versatility of the proposed methodology in adapting to various problems and assumptions, a most challenging recent MTMD robust design example in the literature is chosen for comparison. In this mentioned work, Li and Ni [19] proposed an elegant gradient-based method to design a robustly effective IMTMD considering a desired level of estimation error in structural parameters. For the purpose of comparison under identical assumptions, the assumptions 2–4 have been removed from the model presented in Section 2, and the GA has been made to optimize an irregular 21TMD with a 42-dimensional design space for a structure with 0.02 damping and an uncertain natural frequency. The optimization has been implemented using min max DMF and the criterion (8) with maximum structural frequency error e_R of 0.05 using a population of size 400 through 95 generations. The results have been reported in Figure 15.

Figure 15 depicts the Pareto front of irregular 21TMD in the criterion space. As can be seen, the results by Li and Ni [19] for zero and 0.05 estimation error in the natural frequency of the

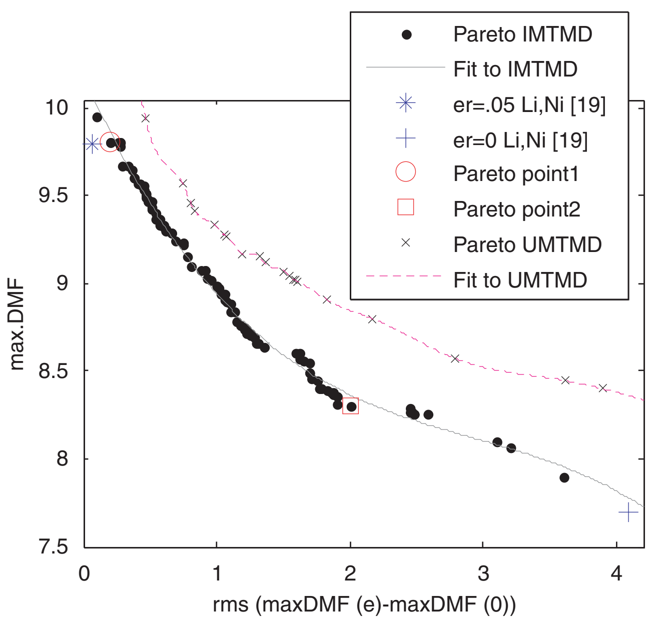

Figure 15. Optimization results for different 21TMDs.

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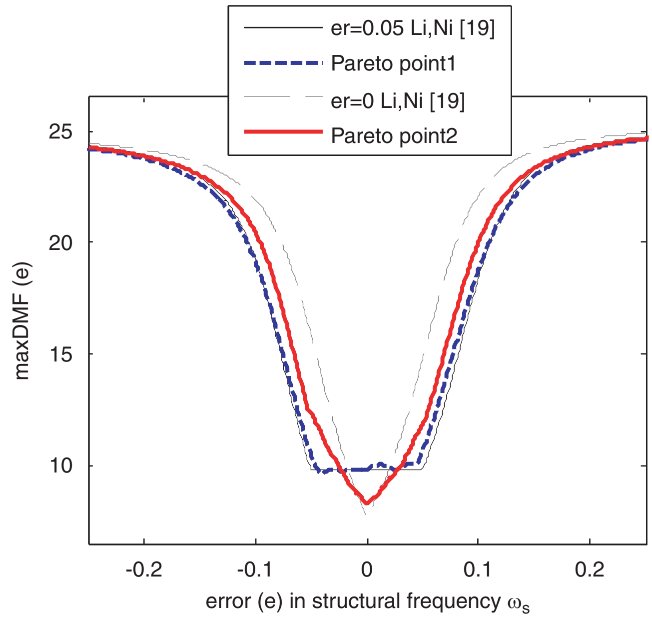

Figure 16. max DMF behavior versus error for different IMTMD designs.

structure might be regarded as vertexes or extremes of the Pareto front obtained in the present investigation. Figure 16 compares the max DMF versus error curves of these two extremes with those of two arbitrary points 1 and 2 in the Pareto set. Despite limitations in the accuracy of the parameters (to maintain an affordable chromosome length), the GA has found a quite satisfactory set of results from which the designer could make an appropriate compromise between effectiveness and robustness, taking advantage of this simple and generic methodology.

For purpose of assessing the performance advantage of irregular MTMD design over uniform design, the Pareto front of uniform 21TMD design (subject to all restrictive assumptions of Section 2) is also depicted in Figure 15. It can be seen that by irregular MTMD design one can achieve dominant results in terms of both effectiveness and robustness. In practice, however, the designer should check if this performance advantage justifies the augmented complexity of implementation.

8. CONCLUSIONS

The main purpose of this paper was to establish a general optimal design methodology for MTMD systems. Toward this purpose, the following procedures and results were respectively carried out and demonstrated:

(1) The optimal design task has been formulated as a two-objective performance-optimization problem. The well-known max DMF index was chosen for effectiveness while for robustness the $\Re$-index was proposed.

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(2) Advantages of exploiting evolutionary heuristics were demonstrated through implementation of two-objective GA optimization capable of converging to the non-dominated set of solutions.
(3) The trade-off between effectiveness and robustness was demonstrated through visualization of Pareto-optimal fronts. This has been especially made apparent by the labeling of selected points in Figures 6 and 7. Moving on the Pareto curve from point 1 to 11 in the design space by increasing the damping and frequency range of the UMTMD, robustness is increased while effectiveness decreases.
(4) A systematic procedure of analyzing the max DMF distribution resulting from structutal frequency under prescribed distribution of error was proposed and carried out to provide validation and further insight into the robustness concept and its trade-off with effectiveness. The min mean (max DMF) criterion was proposed as a preference utility in choosing solutions from the Pareto-optimal set.
(5) It has been shown that the proposed generic methodology has the ability to solve problems under various assumptions. The problem of optimally designing IMTMDs has been considered and solved by the methodology.
(6) The Pareto fronts obtained for optimal effectiveness–robustness design of irregular and uniform MTMDs have been compared and the performance advantage of IMTMD over UMTMD design has been demonstrated. Beside this performance advantage of IMTMD design, however, the practical drawback of increased complexity in manufacturing and implementation should be considered.

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